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Dynamics of currency crises with asset market frictions

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Abstract

This paper presents a dynamic model of currency crises with frictions. By construction, a speculative attack is not an instantaneous event but takes a little time to deplete the country's reserves and, in the event of an attack, agents are uncertain about whether they will be able to act before the devaluation comes. The currency will be overvalued ('ripe for attack') for a long time before an attack takes place. A discrete and sizable devaluation will occur. Small changes in fundamentals may trigger an attack. The model brings insights about the dynamics of currency crises and the effects of some key policy variables.

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1. Introduction

The so-called 'first generation' models of currency crisis (Krugman (1979), Flood and Garber (1984)) show that policies incompatible with a pegged exchange rate regime lead to speculative attacks that produce massive falls in a country's level of reserves and force a government to abandon the peg. Agents attack the currency whenever the "shadow

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exchange rate” exceeds the current exchange rate,¹ a speculative attack instantaneously depletes the country’s reserves. The currency is never overvalued before the attack and discrete jumps in the exchange rate are ruled out.

However, contrary to what those models imply, currencies seems to stay ripe for attack for long periods of time; attacks that take little time to force the abandonment of a pegged regime take much time to start (and sometimes are triggered without major perceived changes in economic fundamentals), and large discrete devaluations are observed.²

In Flood and Garber (1984), discrete devaluations may occur if the “shadow exchange rate” jumps discretely right before the currency attack, but the currency is never overvalued. Other more recent contributions have generated discrete jumps following the abandonment of a peg. Pastine (2002) includes a maximizing government that cares about reserves and does not like speculative attacks in a first generation setup. It shows that the government randomizes the timing of abandoning the peg instead of passively waiting for the agents to attack the currency. So, crisis cannot be predicted and a discrete devaluation occurs because the abandonment of the peg by the Central Bank is not fully expected. In the model of Broner (2001), there is a ‘secular deterioration of fundamentals’ and agents try to guess when the currency will be ‘ripe for attack’. There may be a discrete devaluation because agents are uncertain about the shadow exchange rate. Chamley (2003) also deals with incomplete information and learning: agents are uncertain about whether the mass of speculators is enough to force the Central Bank to abandon the peg. Broner (2004) shows that a discrete devaluation may also occur if some uninformed agents are included in the model. Abreu and Brunnermeier (2003) show how incomplete information can lead to bubbles and crashes. They show that agents may decide to buy an overvalued asset although they know that the bubble will burst at some point in the future. Rochon (2004) applies their argument to currency crises to show that agents delay their attack to the currency.

This paper takes a different approach. Instead of attempting to explain why agents do not perfectly coordinate on attacking the currency whenever it is overvalued, it takes the frictions as its starting point and studies what happens in a dynamic model of currency crises if: (i) an attack does not occurs instantaneously, it takes some (little) time until it forces the abandonment of a peg; and if (ii) agents are uncertain about whether, in the event of an attack, they will be able to escape before the devaluation comes. A simple way to include those features in a model, is to assume that agents get the opportunity of changing position according to a Poisson process, as in Calvo (1983). Due to the Poisson assumption, all agents that are long in the currency at a given point in time face the same probability of being caught by the devaluation.

By modelling frictions in this stylized way, this paper is not explaining why a speculative attack lasts for more than a second — that occurs by assumption. But it is showing that an attack that takes a little time (say, 2 or 3 weeks) to deplete the country’s reserves and force the currency to float will take much more time (say, several months) to get started if the

¹ The shadow exchange rate is what the exchange rate would be if an attack forced the currency to float.

² For example, in the first 3 weeks of December 1994, a strong speculative attack drove *Mexican Peso* to lose a third of its value in a bit more than a week. Following the Russian crisis in August 1998, Brazil lost a third of its foreign reserves in 3 weeks and, in January 1999, *Brazilian Real* lost 40% of its value. In the recent episode in Argentina, the depreciation of the *Peso* was even higher.

agents are uncertain about whether they will be able to escape the devaluation or not. The currency will be “ripe for attack” for a long time, a discrete devaluation will occur and small changes in fundamentals may trigger an attack. The advantage of this approach over a formal modelling of limited participation and/or noisy information about others’ moves that leads to imperfect coordination is its much greater tractability: the effect of the frictions are numerically evaluated, the dynamics of crises under stochastic fundamentals is analyzed and testable implications are offered. The proposed framework provides some insights on the dynamics of currency crisis and on the impacts of frictions, improving macroeconomic prospects and interest rates. Some features of this model often show up in economic and policy analysis but are rarely incorporated in formal models.

The next section presents the non-stochastic model of Flood and Garber (1984). Section 3 adds the Poisson frictions to the model. Section 4 shows that, with uncertainty on the path of shadow exchange rate, the expected devaluation is substantially larger — agents are less aggressive in attacking the peg. A final section concludes.

The model shown at Section 4 can also be seen as a dynamic version of Morris and Shin (1998) with Poisson frictions instead of incomplete information. Like in Morris and Shin (1998), agents are deciding about attacking the currency, the fundamentals are given exogenously and the pressure on the peg depends on other agents’ choices — so expectations play a key role. The structure of the model is very similar to Frankel and Pauzner (2000) model of sectorial choice and the theoretical contribution of Burdzy et al. (2001). Like in Frankel and Pauzner (2000), agents choose between 2 actions and get the opportunity to revise their decisions according to a Poisson process. Payoffs depend on two state variables: a random economic parameter that follows a Brownian motion and the fraction of agents that had chosen one of the actions. However, the assumption of strategic complementarities, key in Morris and Shin (1998) and Frankel and Pauzner (2000), does not hold in the context of this paper. Section 4 further analyzes this issue.

2. Flood and Garber (1984)

In the non-stochastic version of Flood and Garber (1984, FG hereafter), the exchange rate is initially pegged at \bar{S} . Money demand depends negatively on interest rates:

$$\frac{M_t}{P_t} = a_0 - a_1 i_t. \quad (1)$$

Money supply equals foreign currency reserves (R_t) plus domestic credit (D_t).

$$M_t = R_t + D_t. \quad (2)$$

Domestic credit is expanding. FG assumes a linear trend for D_t . Here, it is assumed that D_t grows exponentially.³ So:

$$\frac{\dot{D}_t}{D_t} = \mu_D, \quad \mu_D > 0 \quad (3)$$

³ This change is not important for the results. The main difference is that, in the stochastic version presented at Section 4, the probability distribution of the shadow exchange rate is lognormal instead of normal.

Interest rate parity and purchasing power parity are assumed (i_t^* and P_t^* are constants).

$$i_t = i_t^* + E_t \left(\frac{\dot{S}}{S} \right) \quad (4)$$

$$P_t = S_t \cdot P_t^*. \quad (5)$$

Initially, the government has a positive stock of reserves and will keep the peg until reserves reach a given minimum level (say, until $R_t=0$). In equilibrium, agents know that the peg will be abandoned at time \bar{T} . Before that, $\dot{S}=0$. By PPP (Eq. (5)), $\dot{P}=0$, and by IRP (Eq. (4)), i_t is constant, equal i_t^* . Therefore M_t is also constant (Eq. (1)). Define M^H as the demand for money while the peg is kept:

$$m^H \equiv \frac{M^H}{\bar{S} \cdot P_t^*} = (a_0 - a_1 i_t^*). \quad (6)$$

The expansion of domestic credit generates losses of reserves until the moment in which the peg is abandoned. Then, it leads to an increasing trend in the money supply and, consequently, inflation. Therefore, after the peg is abandoned, the demand for real balances is smaller because the nominal exchange rate is higher, due to inflation (Eqs. (1) and (4)). An arbitrage condition implies that P_t and S_t cannot jump up, and so the discrete reduction in money demand translates in a discrete fall of M_t . Initially, reserves are continuously falling. When R_t is exactly equal to the difference in money demand in both regimes, all agents exchange part of their domestic currency for foreign currency and the government is forced to abandon the peg. Define M^L as the demand for money right after the peg is attacked:

$$m^L \equiv \frac{M^L}{\bar{S} \cdot P_t^*} = \left(a_0 - a_1 \left(i_t^* + \frac{\dot{S}}{\bar{S}} \right) \right). \quad (7)$$

Now, define the shadow exchange rate (\tilde{S}_t) as the exchange rate that would prevail if the currency was allowed to float (demand for real balances would be m^L) and foreign reserves vanished (so that $M_t=D_t$). PPP implies that $P_t=\tilde{S}_t \cdot P_t^*$ and we have:

$$m^L = \frac{M_t}{P_t} = \frac{D_t}{\tilde{S}_t \cdot P_t^*}. \quad (8)$$

Eqs. (7) and (8) imply:

$$\tilde{S}_t = \frac{\bar{S}}{M^L} D_t. \quad (9)$$

The shadow exchange rate (\tilde{S}_t) grows exponentially. Flood and Garber (1984) show that a speculative attack forces the abandonment of the peg exactly when $\tilde{S}_t=\bar{S}$.

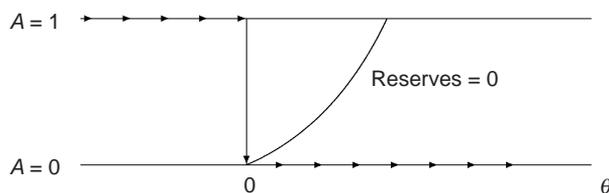


Fig. 1. Schematic representation of Flood and Garber (1984).

2.1. A schematic view of Flood and Garber (1984)

Let A denote the fraction of agents that are holding m_H real balances (and $1 - A$ the fraction of agents holding m_L). Then, during the pegged regime, M_t is given by:

$$M_t = A.M^H + (1 - A)M^L = M^L + A(M^H - M^L).$$

The zero-reserves line is the locus where $M_t = D_t$ so, using Eq. (9), reserves equal zero when:

$$M^L + A(M^H - M^L) = \frac{M^L}{\tilde{S}}$$

Normalizing \tilde{S} and M^L to 1, we get that the peg is abandoned when:

$$A = \frac{\tilde{S} - 1}{M^H - 1}$$

Defining $\theta = \log \tilde{S}$ and $\kappa = M^H - 1$, we get the zero-reserves locus:

$$\theta = \log(1 + \kappa A). \tag{10}$$

The variable θ grows linearly. The behavior of A and θ in FG is represented in Fig. 1.

The behavior of the economy is given by 2 state variables: θ , the (log of the) shadow exchange rate and A , the fraction of the agents that are holding m^H balances.

Flood and Garber (1984) also present a stochastic version of the model. The main message is the same: whenever the currency is overvalued (that is, whenever $\theta > 0$), all agents attack the currency and the peg is abandoned. So, the equilibrium threshold in the stochastic case can also be represented as a vertical line, $\theta^*(A) = 0$, for all A .

3. The model with asset market frictions

3.1. Setup

The model in this section adds frictions and a few technical assumptions to the framework presented above.

There is a continuum of agents, with mass equal to 1, choosing their money balances. In FG, agents' optimal real balance holdings are either m^H or m^L . With frictions, their decisions could be different. For the sake of tractability, the model in this paper restricts their choices to those two possible actions, denoted by *Long* (m^H) and *Not* (m^L).⁴ The fraction of agents that are currently *Long* will be denoted by A . The peg is abandoned when the zero-reserves curve, described by Eq. (10), is reached. We will hereafter denote the zero-reserves function by $\tilde{\theta} = \log(1 + \kappa A)$.⁵ The function $\tilde{\theta}$, as well as the current values of A and θ , are common knowledge. The logarithm of the shadow exchange rate (θ) increases linearly in time at rate μ_θ . Once the peg is abandoned, the exchange rate jumps to its shadow value, $\exp(\theta)$, and the game ends.

In FG, the money demand is given in reduced form by Eq. (1). Here we will assume that there is a constant benefit r for choosing *Long*. However, while in FG higher interest rates decrease the demand for money (because agents hold domestic currency for transactions), in practice the high interest rate differential is a key reason in investors' decision to go long in a country's currency. Thus r will be loosely interpreted as the interest rate differential in our numerical exercises.⁶

The key difference between this model and of FG are the frictions in the asset market. In FG, agents are continuously updating their maximization plans. Here, each agent gets the opportunity to change position according to an independent Poisson process with arrival rate δ , assumed to be greater than r . When the hazard rate of the Poisson clock (δ) approaches infinity, the model in this paper converges to FG: while $\theta \leq 0$, A is 1. Whenever θ gets positive, all agents attack the currency, A jumps to 0, the peg is abandoned and there is no discrete devaluation.⁷

3.2. Equilibrium

Due to the frictions, agents delay their decisions to attack the currency (pick *Not*) and try to 'beat the gun'. This behavior generates a discrete currency devaluation. There is no uncertainty on the path of θ , but luck plays an important role in determining each agent's ex-post payoff.

In this case, although all agents have perfect information about the exchange rate, the timing of the devaluation and its amount, they change position only according to the exogenous Poisson process. Although this setup may not sound very compelling, the

⁴ Besides allowing for greater tractability, this assumption prevents an agent to short arbitrarily high amounts of the domestic currency and trigger a speculative attack by himself.

⁵ The results in this paper do not depend on the functional form of $\tilde{\theta}$ provided it is increasing in A .

⁶ An explicit modelling of interest rate defense, as done in Lahiri and Végh (2003), would substantially complicate the analysis without necessarily adding much to the message of this paper.

⁷ Another caveat of this analysis is that, with frictions, a discrete devaluation will occur. Then, overshooting (or undershooting) could occur, depending on the relative speed of adjustment in the assets and goods markets. Flood and Garber (1984) does not have to deal with this problem, as both markets adjust instantaneously and there are no discrete devaluations. This paper will also disregard such effect. A simple way to keep the connection between this model and FG and avoid this further complication is to assume that, when the devaluation occurs, all agents 'wake up' and all markets adjust instantaneously.

analysis in this section helps us to understand the model in the more interesting and realistic case, when there is uncertainty in the path of θ .

A strategy for the agent yields a decision (*Long* or *Not*) for every pair (θ, A) , that is: $s: R^2 \rightarrow \{Long, Not\}$. A threshold θ^* is a function of A and defines 2 regions: the ‘L’ area, at its left, where agents choose *Long* and the ‘N’ area, at its right, where agents choose *Not*. A threshold equilibrium is a situation in which agents find it optimal to act according to a threshold θ^* (Fig. 2).

The Appendix shows that this model has a unique threshold equilibrium. To characterize the behavior of the system, we need to determine, for each A_0 : $\theta^*(A_0)$ (Θ^* henceforth); the time that it takes for the peg to collapse since θ crosses the threshold (call it ΔT); the value of A when $\tilde{\theta}$ is reached (call it \tilde{A}); and the magnitude of the devaluation $\tilde{\theta}(\tilde{A}) = \log(1 + \kappa\tilde{A})$ ($\tilde{\Theta}$ henceforth).

The time for a devaluation depends on the distance the parameter θ covers since agents start to choose *Not*:

$$\Delta T = \frac{\tilde{\Theta} - \Theta^*}{\mu_\theta}. \tag{11}$$

As $\mu_\theta > 0$, an agent at $\theta^*(A_0)$ knows that everybody will choose *Not* from then on. So, the time for a devaluation is also related to the fraction of agents that is able to run before the devaluation:

$$\tilde{A} = A_0 e^{-\delta \Delta T}. \tag{12}$$

The difference between the expected payoffs of choosing *Long* and *Not* equals the expected payoff of going long in the currency until the next opportunity of choosing — what happens after that is irrelevant because the present choice has no influence in future decisions. An agent that is long in the currency gets $\exp(r \cdot dt)$ at every dt if the peg has not been abandoned yet. But when the devaluation comes, his balance is multiplied by $\exp(-\theta)$. The only source of uncertainty is the realization of the Poisson process, and the expected payoff of choosing *Long* is given by:

$$E\pi = \int_0^{\Delta T} \delta e^{-\delta t} e^{rt} dt + e^{r\Delta T} e^{-\tilde{\theta}} \int_{\Delta T}^\infty \delta e^{-\delta t} dt - 1.$$

The first term is what an agent gets if he receives a signal before time ΔT . The second term is the agent’s return if he is caught by the devaluation. Doing the algebra,

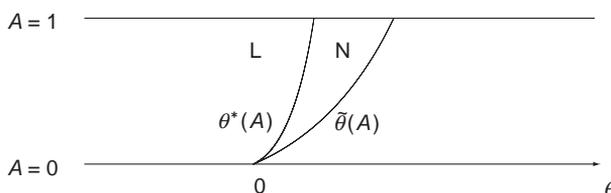


Fig. 2. Investor’s threshold for investing ($\theta^*(A)$).

we obtain:

$$E\pi = \left(1 - e^{-(\delta-r)\Delta T}\right) \frac{\delta}{\delta - r} + e^{(r-\delta)\Delta T - \tilde{\Theta}} - 1. \tag{13}$$

Making $E\pi=0$, we get ΔT :

$$\Delta T = \frac{\log\left(\frac{\delta}{r}\left(1 - e^{-\tilde{\Theta}}\right) + e^{-\tilde{\Theta}}\right)}{\delta - r}. \tag{14}$$

Eqs. (12) and (14) yield:

$$\frac{\tilde{A}}{A_0} = \left(\frac{r}{re^{-\tilde{\Theta}} + \delta(1 - e^{-\tilde{\Theta}})}\right)^{\frac{\delta}{\delta-r}}. \tag{15}$$

The function $\tilde{\Theta} = \log(1 + \kappa\tilde{A})$ and Eq. (15) yield \tilde{A} and we can easily back out all other parameters.

Let $\psi = \delta/r$. Then, the Eq. (15) becomes:

$$\frac{\tilde{A}}{A_0} = \left(\psi\left(1 - e^{-\tilde{\Theta}}\right) + e^{-\tilde{\Theta}}\right)^{\frac{-\psi}{\psi-1}}. \tag{16}$$

The next proposition presents the main properties of the equilibrium.

Proposition 1. *When $\mu_\theta > 0$ and $\sigma_\theta = 0$:*

1. $\tilde{\Theta}$ and \tilde{A} are increasing in r .
2. $\tilde{\Theta}$ and \tilde{A} are decreasing in δ .
3. As $\psi \rightarrow \infty$, this model converges to Flood and Garber (1984):
 $\lim_{\psi \rightarrow \infty} \tilde{A} = \lim_{\psi \rightarrow \infty} \tilde{\Theta} = \lim_{\psi \rightarrow \infty} \Delta T = \lim_{\psi \rightarrow \infty} \Theta^* = 0$.
4. $\tilde{\Theta}$ and \tilde{A} depend on $\psi = \delta/r$, but, for a given ψ , do not depend on δ or r .

Proof: see Appendix.

Corollary 1. *For any $r > 0$ and $\delta < \infty$, \tilde{A} and $\tilde{\Theta}$ are strictly positive.*

The higher is r , the higher are the incentives for an agent to go long in the currency and face the risk of not being able to run before the crowd. Moreover, higher frictions in the asset market (i. e. lower values of δ) imply a higher depreciation. One could think that if agents had less frequent opportunities of choosing (δ was lower), they would have an incentive to withdraw their money earlier, and so depreciation would come sooner. However, a lower δ also implies that *other agents* will take more time to change position, so it does not reduce the odds of escaping. The important effect of a reduction in δ is the increase in the time an attack takes (ΔT) and, therefore, in the interest got during this period. Loosely speaking, the benefit of going long in the currency is proportional to $1/\delta$. Indeed, as Eq. (16) shows, \tilde{A} and $\tilde{\Theta}$ depend only on the ratio $\Psi = \delta/r$, the impacts of increases in $1/\delta$ and in r on agents' behavior are exactly the same. When δ is finite, there

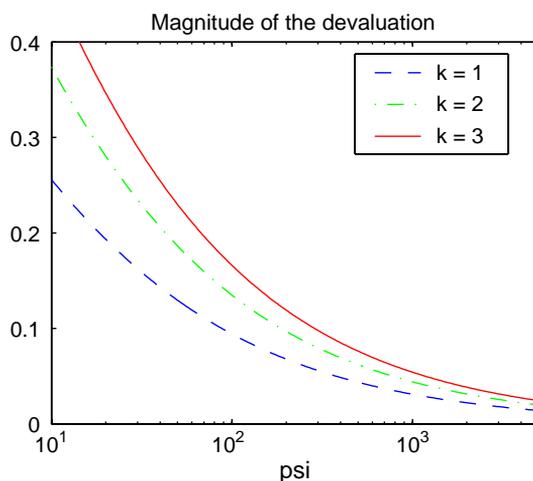


Fig. 3. $\tilde{\Theta} \times \psi$.

is a discrete jump in the exchange rate ($\tilde{\Theta} > 0$) and a positive number of agents lose money ($\tilde{A} > 0$).

Fig. 3 shows the magnitude of a devaluation ($\tilde{\Theta}$) as a function of ψ for some values of κ and $A_0=1$. The function $\tilde{\Theta}$ is given by Eq. (10), so setting κ equal to 1, 2 and 3 means that the exchange rate would jump to, respectively, 2, 3 and 4 if all agents kept holding the domestic currency until the government could not sustain the peg anymore. We can see that $\tilde{\Theta}$ do not decay slowly: for $r=1\%$ a month, $\delta=20$ (one signal for each business day, on average) and $\kappa=1$, we have $\psi=2000$, a magnitude of devaluation of 2.2%, which is the interest achieved in 2.2 months, and a speculative attack takes less than 0.2 months to force the government to leave the peg. If agents get on average one signal a week ($\delta=4$), $r=3\%$ a month and $\delta=3$, we get $\psi=133$, the magnitude of a devaluation is 14.5% and the attack takes around 3 weeks to force the government to abandon the peg.

The parameter μ_θ has no effect on any variable but θ^* . It is clear from Eq. (11) that a higher μ_θ leads to a lower θ^* : the agents' willingness to choose *Long* depends negatively on the rate currency overvaluation is expected to increase. For very big values of μ_θ , the curve θ^* may depend negatively on A_0 for (at least) some values of A_0 .

4. Adding uncertainty to the path of θ

This section adds uncertainty to the path of the shadow exchange rate by assuming that θ follows a Brownian motion:

$$d\theta = \mu_\theta dt + \sigma_\theta dX.$$

In the model of Section 3, agents know that after a speculative attack starts all agents will chose *Not*. Here it is shown that when θ follows a Brownian motion, a speculative attack can be reversed, so there are more incentives to hold the domestic currency and the expected magnitude of a devaluation is higher.

The qualitative implications of the model in this section do not depend on a positive trend for μ_θ . All results hold with zero or negative π_θ . In particular, when agents have arbitrarily frequent opportunity of changing position ($\delta \rightarrow \infty$), a speculative attack instantaneously depletes the government's stock of foreign reserves whenever $\theta > 0$, leading to the abandonment of the peg with no discrete devaluation. A reasoning similar to the backward induction argument of Krugman (1979) and Flood and Garber (1984) applies.

4.1. The agent's problem

The occurrence and size of a devaluation depend on others' decisions and on a stochastic shadow exchange rate. To make a decision, an agent needs to forecast what others will do in all possible situations in order to estimate his expected payoff of holding the currency. Let $(\theta_0; A_0)$ denote the current state of the economy. Let z denote a particular realization of the Brownian motion. Call $\Delta t(z)$ the time it will take for the devaluation and $\theta^{\text{end}}(z)$ the size of the devaluation. Suppose agents act according to a threshold θ^* as defined at Section 3. Given z and θ^* , the only source of uncertainty is the realization of the Poisson process and Eq. (13) now becomes:

$$\pi(z; \theta_0, A_0, \theta^*) = \left(1 - e^{-(\delta-r)\Delta t(z)}\right) \frac{\delta}{\delta - r} + e^{-(\delta-r)\Delta t(z) - \theta^{\text{end}}(z)} - 1. \quad (17)$$

Given that all agents choose according a threshold θ^* , Long is the optimal choice if $E\pi(\theta_0, A_0, \theta^*) \geq 0$ where:

$$E\pi(\theta_0, A_0, \theta^*) = \int_z \pi(z; \theta_0, A_0, \theta^*) f(z) dz. \quad (18)$$

4.2. Equilibrium

The structure of this model is very similar to Frankel and Pauzner (2000). However, the conditions for the existence of a unique rationalizable equilibrium required by their theorem do not hold in this model. Frankel and Pauzner (2000) assume that agents' actions are strategic complements: the expected payoff of choosing one action is increasing in the fraction of agents expected to choose the same action in the future. The property of strategic complementarities is also key in the models following Morris and Shin (1998).⁸ One could think that such property would also hold in the context of this model. It does not. It is true that the likelihood of a devaluation in the near future depends negatively on how many agents decide to go long in the currency, as in Morris and Shin (1998). However, the expected magnitude of a devaluation is increasing on the fraction of agents expected to go long in the currency: if agents are less aggressive in attacking the currency, the devaluation, when it occurs, will be higher. The speculative attack works as a discipline device, preventing the currency from being more overvalued. In some cases, the effect in the magnitude dominates the effect in the probability: it is possible to construct

⁸ See Morris and Shin (2003) for a survey.

examples in which an agent would choose *Long* if all others were not doing so but would choose *Not* if all other agents were expected to choose *Long*.

The effect of agents' decisions in the magnitude of a devaluation is a natural result in the dynamic framework of this paper but does not occur in the static setup of Morris and Shin (1998). Here, as in Abreu and Brunnermeier (2003), there is not only *coordination* but also *competition* among agents. Morris and Shin (1998) capture the coordination issue but not the preemptive motivations of investors.⁹

Although it is not possible to apply Frankel–Pauzner tools to obtain a unique rationalizable equilibrium, it can be shown that there exists a unique equilibrium within the class of *threshold equilibria*. The proof relies on the following assumptions:¹⁰

- Dominant regions: it is optimal to choose *Long* when the economy is too far from the threshold $\tilde{\theta}$ and *Not* when it is too close.
- Agents face the trade-off between a constant flow benefit of choosing one action and a negative discrete cost, that occurs if the threshold $\tilde{\theta}$ is attained. The threshold $\tilde{\theta}$ is increasing in A and the discrete loss (the devaluation) is increasing in θ .¹¹ No other assumptions on the functional forms of $\tilde{\theta}$ and the discrete loss are required.
- Due to the Poisson frictions, all information about the current state of the economy is given by the current values of θ and A and the future path of the economy depends only on the current state, the threshold θ^* and the realization of the Brownian motion. In particular, when the economy is at the threshold, the expected payoff for all agents from then on is 0 — as if the game was over or as if there was a devaluation of size 0.

4.3. The effect of uncertainty about the path of θ

If $\sigma > 0$, analytical solutions are not available but an approximated value for the threshold $\theta^*(A)$, for specific parameters δ , r , μ_θ , σ_θ and κ , can be found using numerical methods. The task consists in finding a function $\theta^*(A)$ that solves a discrete version of the model, with θ following a random walk instead of a Brownian motion.

Fig. 4 shows the line $E\pi=0$ conditional on every agent choosing *Not* from then on (curve 1) and the equilibrium threshold (curve 2). The curve 1 is almost identical to the threshold for the case with no uncertainty on the path of θ described in Section 3.

The difference between the curves 1 and 2 can be seen as the effect of the possibility that others will choose *Long* later. For any positive μ_θ , the 'L' area is larger if there is some (not-too-big) degree of uncertainty on θ (as opposed to $\sigma_\theta=0$) because that generates the possibility that others will choose *Long* in the future.

In the model presented at Section 3, the agent chooses *Long* despite an attack is eminent because he has good chances of escaping before the devaluation comes. At the left of curve 1, the possibility of getting another signal before the zero-reserves line is reached

⁹ Such essentially dynamic features are also absent in the dynamic model of currency crisis in Morris and Shin (1999).

¹⁰ The proof is available at <http://personal.lse.ac.uk/guimarae/>.

¹¹ If the loss due to a devaluation was exogenous and independent of θ , the argument in Frankel and Pauzner (2000) could be applied here.

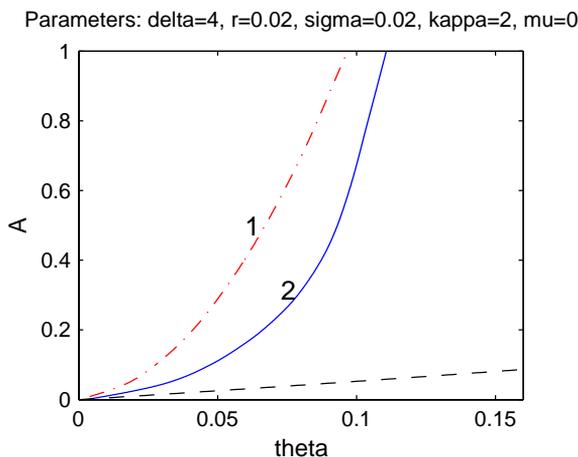


Fig. 4. Effect of uncertainty.

is enough for encouraging an agent to go long in the currency. With uncertainty, the attack may be reversed: if θ decreases enough and crosses the threshold θ^* , all agents start to choose *Long* again and the peg is not abandoned. So, incentives for choosing *Long* are significantly stronger when there is uncertainty on the path of θ .

4.4. Example

The following example illustrates the mechanics of the model. Fig. 5 brings the threshold θ^* and an example of the path of (θ, A) . In the example, the unit of time is a month, $\delta=4$, $r=0.02$, $\mu_\theta=0$, $\sigma_\theta=0.02$ and $\kappa=2$. The parameter δ is related to the time an attack takes to force the abandonment of the peg, in this case it takes on average 3 weeks. An interest rate differential of 2% a month as the *maximum* the government would pick in order to defend the peg sounds reasonable (less than in Brazil, 1997–1998). Setting $\kappa=2$ means that if all agents kept holding the domestic currency regardless of the value of θ , the exchange rate after the devaluation would be 3 (less than the observed devaluation in Argentina, 2002).

The system starts at the point indicated by a circle ($\theta_0=0.11$, $A_0=0.9$). All agents choose *Not* and A drops to around 0.58 in just 3.5 days. But then, x crosses θ^* , gets to the ‘L’ area and everybody starts to go long in the currency again. Months later, A is very close to 1 and the economy crosses the threshold θ^* to the ‘N’ side. In 3 weeks, $\tilde{\theta}$ is reached and the peg is gone. The exchange rate jumps to $e^{\theta^{\text{end}}}$; that is, to around 1.14. 93.5% of the agents are able to escape before the devaluation comes. Although the numerical example should be taken with caution, it is worth mentioning that this devaluation size is similar to the estimates of the *expected* magnitude of a devaluation in Brazil, reflected in the option prices, presented in Guimarães (2004).

Ex-post, the payoff of a particular agent depends on luck: some will get all the benefits of being long in the currency and pick *Not* right before the crisis, others will withdraw earlier, others will be caught by the devaluation, and so on.

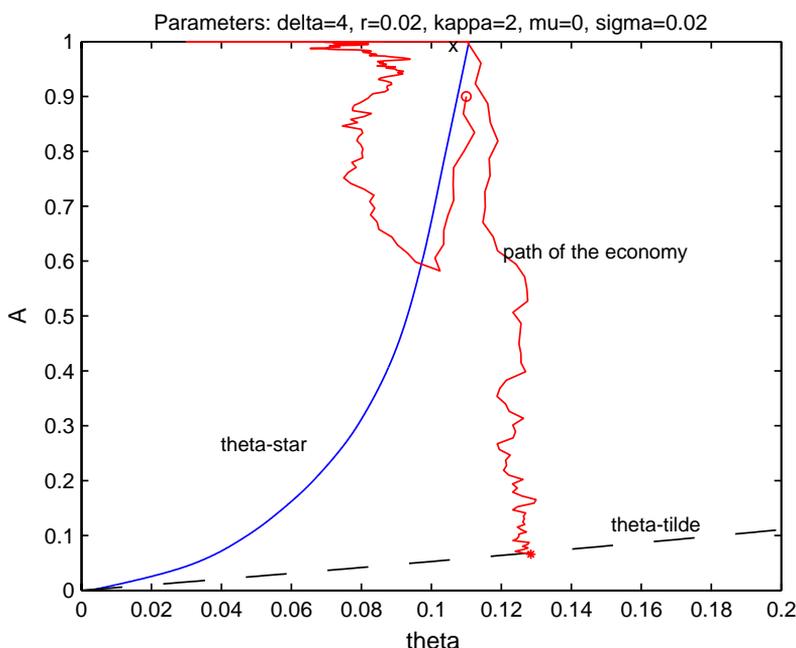


Fig. 5. Example of the dynamics of the model.

An agent that decided right before the threshold was crossed at the second time, at the point indicated by an ‘x’ in Fig. 5, chose *Long* for 2 reasons: (i) in the case of a ‘good’ realization of z from then on, agents would keep choosing *Long* for a long time, as they had done for months and there would be no crisis; (ii) in the case of a ‘bad’ realization of z , the agent could still get the signal and run before the crisis, as most of the agents did. In the 3 weeks preceding the devaluation, the interest earned was below 1.5%, much less than the depreciation of 14%, but good enough for encouraging agents deciding at the ‘L’ area to take the risk.

Sometimes, large speculative attacks follow small changes in economic variables. As Obstfeld and Rogoff (1995) pointed, “the speculative attack on the British pound in September 1992 would certainly have succeeded had it occurred in August — so why did speculators wait?”.¹² In the model, a speculative attack would certainly succeed if it had started at the point indicated with an ‘x’ or even before. Speculators wait because it pays off to face the risk. In equilibrium, a crisis may be triggered by a small change in θ , if it pushes the shadow exchange rate beyond the threshold θ^* .

Countries usually resist for quite a long time before abandoning their pegs and many agents are able to escape from the crisis with little or no loss. Many times, the country wins the battle and the peg is not abandoned (in the example, that happened in the very first days, when A dropped from 0.9 to 0.58). Agents who decide not to withdraw their money in such cases make profits.

¹² Page 86.

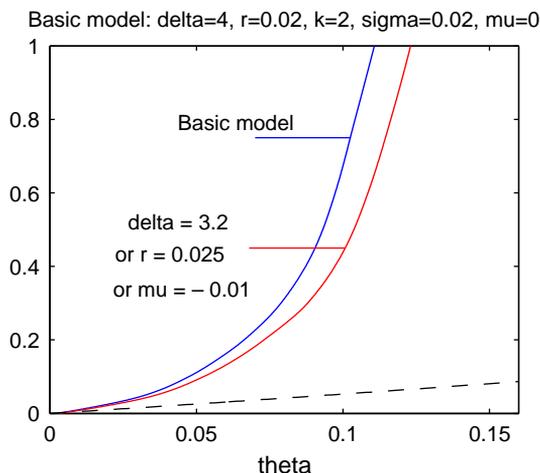


Fig. 6. Effects of changing exogenous variables.

4.5. The effect of exogenous parameters

The model shows that the agents' willingness to go long in the currency, for a given level of θ and A , increases on: (i) frictions, (ii) interest rates, and (iii) the rate currency overvaluation tends to decrease. So, raising any of the above variables would reduce the probability of a crisis but would also *increase* the expected magnitude of a currency devaluation, conditional on its occurrence.¹³ Fig. 6 shows the equilibrium threshold for some given parameters $-\mu\theta=0$, $\delta=4$, $r=0.02$, $\kappa=2$ and $\sigma_\theta=0.02$ — and the curve θ^* when one of those parameter changes. We see that the effect of decreasing μ_θ (the shadow exchange rate trend) from 0 to -0.01 is similar to the impact of changing δ from 4 to 3.2 or r from 0.02 to 0.025: $\theta^*(1)$ goes from 0.1108 to around 0.123. Note that having $\mu_\theta=-0.01$ implies that the expected θ in a year is around zero if the economy is close to $\theta^*(1)$.

One could think that so large change in μ_θ would push the threshold far to the right because agents would attribute high probability for others choosing *Not* in the future, which could lead to a chain effect and have major effects on θ^* . That does not happen because, in the very short run, the stochastic component of the Brownian motion is more important than the trend and movements in A are even more dramatic. As an attack takes little time to deplete the country's reserves and force a devaluation, nothing but the near future matters in this game. The perspective of the economy in a year is not relevant for an agent deciding in the middle of the turmoil. That generates room for policies aiming at helping a country to survive a crisis conditional on improving macroeconomic conditions.

¹³ Those results were shown analytically for the model in Section 3, with no uncertainty on the path of θ . The simulation results allow us to conjecture that such comparative statics also hold when μ follows a Brownian motion for empirically plausible parameter values.

Raising interest rates (parameter r) could affect θ and μ_θ , the shadow–exchange–rate trend. Disregarding such effects, the model shows a positive impact of interest rates on the agents' willingness to choose *Long*. Such conclusion sounds natural but may not be obtained in a standard multiple equilibrium setting. Furman and Stiglitz (1998), for example, argue that the payoff of going long is of order of dt if the peg survives but is large and negative if the peg is abandoned. So, only unrealistic interest rates would be able to defend the currency. The missing piece in their analysis are the asset market frictions. Indeed, governments usually raise interest rates when a speculative attack starts in order to defend their pegs and, sometimes, they succeed (as Brazil in 1997, for example).

The parameter r is assumed to be constant. Now, call $\theta^*(r)$ the equilibrium threshold for a given r and suppose that the benefit of holding the domestic currency is not r but $R(\theta, A)$. It is easy to find functions $R(\theta, A)$ such that (i) at all points at the left of $\theta^*(r)$, *Long* would be the optimal action, and (ii) at all points at the right of $\theta^*(r)$, *Not* would be optimal. Thus, acting according to $\theta^*(r)$ would be an equilibrium if r was replaced by some function $R(\theta, A)$ satisfying (i) and (ii). At the 'L'-side, very close to $\theta^*(r)$, $R(\theta, A)$ would have to be very close to r , so that agents would still find it optimal to choose *Long*, but further left of the threshold, $R(\theta, A)$ could be much smaller. Thus, we could interpret r as the maximum interest rate observed in this economy.

4.6. A testable implication

The model presented in this paper yields a distinguishable testable implication. It predicts that a speculative attack is triggered when the shadow exchange rate hits a threshold. Therefore, the probability of a devaluation varies according to the shadow exchange rate — it depends on how far the economy is from the threshold. The expected magnitude of a devaluation, conditional on its occurrence, is relatively stable, because agents know that when the speculative attack starts, the shadow exchange rate will be around the threshold. Guimarães (2004) finds some empirical support for this implication using data on options of Brazilian exchange rate.

5. Concluding remarks

In a major speculative attack following the Russian crises, in 1998, Brazil lost a third of its foreign reserves in around 3 weeks. For many practical purposes, this is close enough to an instantaneous attack, but this paper shows that new insights arise in a model in which an attack is not an instantaneous event but lasts for a few weeks.

Given the complexity of real world problems and the difficulties related to obtaining and analyzing information, it is indeed not surprising that agents do not act exactly at the same time. Without denying that an explicit modelling of informational issues may teach us important lessons, this paper argues that including such frictions in a stylized way in our theoretical models also has important benefits for allowing us to focus on the economic problems we are really interested in.

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Appendix A. Proofs

A.1. Existence and uniqueness of equilibrium in the model of Section 3

If $\mu_0 > 0$, there exist an area in which *Long* is the optimal choice regardless what others do and another area in which *Not* is the optimal decision in any case. Thus, there cannot be any equilibrium in which people choose one of the actions regardless of the state of the economy.

If Eq. (16) defines a unique curve θ^* such that agents' expected payoff equals 0 at all its points, then *Long* is the optimal choice at the left of θ^* and *Not* is the optimal choice at its right, because the time for a devaluation and its size are deterministic functions of the threshold and the current state. To see that for any $A_0 \in [0, 1]$ there exists a unique \tilde{A} satisfying Eq. (16), note that: (i) its left hand side is increasing in \tilde{A} and its right hand side is decreasing in \tilde{A} – as $\tilde{\theta}$ is increasing in A – and (ii) its right hand side assumes values inside the (0, 1)-interval and its left hand side equals 0 if $\tilde{A}=0$ and 1 if $A=A_0$.

$$\frac{\tilde{A}}{A_0} = \left(\psi \left(1 - e^{-\tilde{\theta}} \right) + e^{-\tilde{\theta}} \right)^{\frac{-\psi}{\psi-1}} . \tag{Equation 16}$$

A.2. Proof of Proposition 1

Proof of Statements 1 and 2. It's clear that \tilde{A} must lie in the interval (0, 1). The agents that choose right before the devaluation must prefer *Not*, so \tilde{A} must be smaller than 1. ΔT must be finite, so \tilde{A} must be greater than 0.

To show that $(d\tilde{A}/d\delta) < 0$ and $(d\tilde{A}/dr) > 0$, we need to show that $(d\tilde{A}/d\psi) < 0$. Taking logs of Eq. (16), we get:

$$\frac{\psi - 1}{\psi} \log \left(\frac{\tilde{A}}{A_0} \right) = - \log \left(e^{-\tilde{\theta}} + \psi \left(1 - e^{-\tilde{\theta}} \right) \right). \tag{19}$$

Differentiating Eq. (19) with respect to ψ , we get:

$$\left[\frac{\psi - 1}{\tilde{A}\psi} + \frac{(\psi - 1)e^{-\tilde{\theta}} \left(\frac{d\tilde{\theta}}{d\psi} \right)}{e^{-\tilde{\theta}} + \psi(1 - e^{-\tilde{\theta}})} \right] \frac{d\tilde{A}}{d\psi} = - \frac{\log \left(\frac{\tilde{A}}{A_0} \right)}{\psi^2} - \frac{\left(1 - e^{-\tilde{\theta}} \right)}{e^{-\tilde{\theta}} + \psi(1 - e^{-\tilde{\theta}})}. \tag{20}$$

Substituting Eq. (19) in (20), we get:

$$\left[\frac{\psi - 1}{\tilde{A}\psi} + \frac{e^{-\tilde{\theta}}(\psi - 1)\left(\frac{d\tilde{\theta}}{d\tilde{A}}\right)}{e^{-\tilde{\theta}} + \psi(1 - e^{-\tilde{\theta}})} \right] \frac{d\tilde{A}}{d\psi} = \frac{\log\left(e^{-\tilde{\theta}} + \psi(1 - e^{-\tilde{\theta}})\right)}{\psi(\psi - 1)} - \frac{(1 - e^{-\tilde{\theta}})}{e^{-\tilde{\theta}} + \psi(1 - e^{-\tilde{\theta}})}$$

As $\left[e^{-\tilde{\theta}} + \psi(1 - e^{-\tilde{\theta}})\right] > 1$,

$$\log\left(e^{-\tilde{\theta}} + \psi(1 - e^{-\tilde{\theta}})\right) < \left(e^{-\tilde{\theta}} + \psi(1 - e^{-\tilde{\theta}}) - 1\right)$$

and so:

$$\left[\frac{\psi - 1}{\tilde{A}\psi} + \frac{e^{-\tilde{\theta}}(\psi - 1)\left(\frac{d\tilde{\theta}}{d\tilde{A}}\right)}{e^{-\tilde{\theta}} + \psi(1 - e^{-\tilde{\theta}})} \right] \frac{d\tilde{A}}{d\psi} < \frac{\left(e^{-\tilde{\theta}} + \psi(1 - e^{-\tilde{\theta}}) - 1\right)}{\psi(\psi - 1)} - \frac{(1 - e^{-\tilde{\theta}})}{e^{-\tilde{\theta}} + \psi(1 - e^{-\tilde{\theta}})}$$

Simplifying,

$$\left[\frac{\psi - 1}{\tilde{A}\psi} + \frac{e^{-\tilde{\theta}}(\psi - 1)\left(\frac{d\tilde{\theta}}{d\tilde{A}}\right)}{e^{-\tilde{\theta}} + \psi(1 - e^{-\tilde{\theta}})} \right] \frac{d\tilde{A}}{d\psi} < - \frac{(1 - e^{-\tilde{\theta}})(\psi - 1)e^{-\tilde{\theta}}}{\psi(e^{-\tilde{\theta}} + \psi(1 - e^{-\tilde{\theta}}))} < 0$$

which yields: $(d\tilde{A}/d\psi) < 0$. □

Proof of Statement 3. Taking the limit of Eq. (14), we get:

$$\lim_{\delta \rightarrow \infty} \Delta T = 0.$$

Moreover, suppose $\delta \rightarrow 1$ and $\tilde{\Theta} = \varepsilon$, bounded away from 0. Then, by Eq. (15), $\tilde{A} \rightarrow 0$. But that implies $\tilde{\Theta} = 0$, a contradiction. Thus:

$$\lim_{\delta \rightarrow \infty} \tilde{\Theta} = 0 \Rightarrow \lim_{\delta \rightarrow \infty} \tilde{A} = 0$$

From Eq. (11), $\lim_{\delta \rightarrow \infty} \Theta^* = 0$. □

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